Department of Mechanical Engineering

DESIGN OF MACHINE MEMBERS-I (R16)

UNIT – I

INTRODUCTION: General considerations in the design of Engineering Materials and their properties – selection –Manufacturing consideration in design, tolerances and fits –BIS codes of steels.
STRESSES IN MACHINE MEMBERS: Simple stresses – combined stresses – torsional and bending stresses – impact stresses – stress strain relation – various theories of failure – factor of safety – design for strength and rigidity – preferred numbers. the concept of stiffness in tension, bending, torsion and combined situations – static strength design based on fracture toughness.

Define machine design?
Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

Explain the Classifications of Machine Design
The machine design may be classified as follows:
1. Adaptive design. In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.
2. Development design. This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.
3. New design. This type of design needs lot of research, technical ability and creative think- ing. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design.
The designs, depending upon the methods used, may be classified as follows :
(a) Rational design. This type of design depends upon mathematical formulae of principle of mechanics.
(b) Empirical design. This type of design depends upon empirical formulae based on the practice and past experience.
(c) Industrial design. This type of design depends upon the production aspects to manufacture any machine component in the industry.
(d) Optimum design. It is the best design for the given objective function under the specified constraints. It may be achieved by minimising the undesirable effects.
(e) System design. It is the design of any complex mechanical system like a motor car.
(f) Element design. It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.
(g) Computer aided design. This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimisation of a design.

Explain the manufacturing consideration in design?

General Considerations in Machine Design

Following are the general considerations in designing a machine component: ***1. Type of load and stresses caused by the load.*** The load, on a machine component, may act in several ways due to which the internal stresses are set up.

2. Motion of the parts or kinematics of the machine. The successful operation of any ma- chine depends largely upon the simplest arrangement of the parts which will give the motion required. The motion of the parts may be :

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions. (*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic. (*c*) Constant velocity. (*d*) ***Constant*** or variable acceleration. **3. *Selection of materials.*** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behaviour under working conditions. Some of the important characteristics of materials are : strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, machinability, electrical conductivity, etc.

4. *Form and size of the parts.* The form and size are based on judgement. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is neces- sary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

5. *Frictional resistance and lubrication.* There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings**.**

6. *Convenient and economical features.* In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take- up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible.

The economical operation of a machine which is to be used for production, or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

7. *Use of standard parts.* The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order.

The standard or stock parts should be used whenever possible ; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

8. *Safety of operation.* Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

9. *Workshop facilities.* A design engineer should be familiar with the limitations of his employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

10. *Number of machines to be manufactured.* The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

11. *Cost of construction.* The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of hand made samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions, should be to reduce the manufacturing cost to the minimum. **12. *Assembling.*** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

Explain the general procedure in Machine Design

General Procedure in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows :

1. *Recognition of need.* First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. *Synthesis* (*Mechanisms*). Select the possible mechanism or group of mechanisms which will give the desired motion.

3. *Analysis of forces.* Find the forces acting on each member of the machine and the energy transmitted by each member.

4. *Material selection.* Select the material best suited for each member of the machine.

5. *Design of elements* (*Size and Stresses*). Find the size of each member of the machine by considering the force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component, as per the drawing, is manufactured in the workshop. The flow chart for the general procedure in machine design

How do you classify materials for engineering use?

Classification of Engineering Materials

The engineering materials are mainly classified as :

1. Metals and their alloys, such as iron, steel, copper, aluminium, etc.

2. Non-metals, such as glass, rubber, plastic, etc. The metals may be further classified as : (*a*) Ferrous metals, and (*b*) Non-ferrous metals

The **ferrous metals*** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

The ***non-ferrous*** metals are those which have a metal other than iron as their main constituent, such as copper, aluminium, brass, tin, zinc, etc.

What are the factors to be considered for the selection of materials for engineering purpose? Discuss.

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material :

1. Availability of the materials, **2.** Suitability of the materials for the work- ing conditions in service, and **3.** The cost of the materials. The important properties, which determine the utility of the material are physical, chemical and mechanical properties.

Physical Properties of Metals

The physical properties of the metals include luster, colour, size and shape, density, electric and thermal conductivity, and melting point.

Define 'mechanical property' of an engineering material. State any six mechanical properties, give their definitions and one example of the material possessing the properties or List the mechanical properties.

Mechanical Properties of Metals

1. Strength. It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called *stress.

2. Stiffness. It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

3. Elasticity. It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

4. Plasticity. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. Ductility. It is the property of a material enabling it to be drawn into wire with the applica- tion of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminium, nickel, zinc, tin and lead.

6. *Brittleness.* It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads, snap off without giving any sensible elongation. Cast iron is a brittle material.

7. *Malleability.* It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminium.

8. *Toughness.* It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed upto the point of fracture. This property is desirable.

9. Machinability. It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove the material at some given rate or the energy required to remove a unit volume of the material.

10. Resilience. It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

11. *Creep.* When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called ***creep.*** This property is considered in designing internal combustion engines, boilers and turbines.

12. *Fatigue.* When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses.

13. Hardness. It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal.

Briefly explain the BIS codes of steel.

Steels Designated on the Basis of Mechanical Properties

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard these steels are designated by a symbol 'Fe'or 'Fe E'depending on whetherthe steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm^2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm^{2}and 'Fe E 220' means a steel having yield strength of 220 N/mm^2

Notes : 1. The steels from grades Fe 290 to Fe 490 are general structural steels and are available in the form of bars, sections, tubes, plates, sheets and strips.

2. The steels of grades Fe 540 and Fe 620 are medium tensile structural steels.
3. The steels of grades Fe 690, Fe 770 and Fe 870 are high tensile steels.

Steels Designated on the Basis of Chemical Composition

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

(*a*) Figure indicating 100 times the average percentage of carbon content,

(*b*) Letter 'C', and

(*c*) Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on an average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

Free Cutting Steels

The free cutting steels contain sulphur and phosphorus. These steels have higher sulphur content than other carbon steels. In general, the carbon content of such steels vary from 0.1 to 0.45 per cent and sulphur from 0.08 to 0.3 per cent. These steels are used where rapid machining is the prime requirement. It may be noted that the presence of sulphur and phosphorus causes long chips in machining to be easily broken and thus prevent clogging of machines. Now a days, lead is used from 0.05 to 0.2 per cent instead of sulphur, because lead also greatly improves the machinability of steel without the loss of toughness.

According to Indian standard, IS : 1570 (Part III)-1979 (Reaffirmed 1993), carbon and carbon manganese free cutting steels are designated in the following order :

1. Figure indicating 100 times the average percentage of carbon,
2. Letter 'C',
3. Figure indicating 10 times the average percentage of manganese, and
4. Symbol 'S' followed by the figure indicating the 100 times the average content of sulphur. If instead of sulphur, lead (Pb) is added to make the steel free cutting, then symbol 'Pb' may be used.

Indian Standard Designation of Low and Medium Alloy Steels

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), low and medium alloy steels shall be designated in the following order :

1. Figure indicating 100 times the average percentage carbon.
2. Chemical symbol for alloying elements each followed by the figure for its average percentage content multiplied by a factor as given below :

Element	*Multiplying factor*
Cr, Co, Ni, Mn, Si and W	4
Al, Be, V, Pb, Cu, Nb, Ti, Ta, Zr and Mo	10
P, S and N	100

For example 40 Cr 4 Mo 2 means alloy steel having average 0.4% carbon, 1% chromium and 0.25% molybdenum.

Notes : 1. The figure after multiplying shall be rounded off to the nearest integer.

2. Symbol 'Mn' for manganese shall be included in case manganese content is equal to or greater than 1 per cent.

3. The chemical symbols and their figures shall be listed in the designation in the order of decreasing content.

Indian Standard Designation of High Alloy Steels (Stainless Steel and Heat Resisting Steel)

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), the high alloy steels (*i.e.* stainless steel and heat resisting steel) are designated in the following order:

1. Letter 'X'.
2. Figure indicating 100 times the percentage of carbon content.
3. Chemical symbol for alloying elements each followed by a figure for its average percentage content rounded off to the nearest integer.
4. Chemical symbol to indicate specially added element to allow the desired properties.

For example, X 10 Cr 18 Ni 9 means alloy steel with average carbon 0.10 per cent, chromium 18 per cent and nickel 9 per cent.

Indian Standard Designation of High Speed Tool Steel

According to Indian standard, IS : 1762 (Part I)-1974 (Reaffirmed 1993), the high speed tool steels are designated in the following order :

1. Letter 'XT'.
2. Figure indicating 100 times the percentage of carbon content.
3. Chemical symbol for alloying elements each followed by the figure for its average percentage content rounded off to the nearest integer, and
4. Chemical symbol to indicate specially added element to attain the desired properties.

For example, XT 75 W 18 Cr 4 V 1 means a tool steel with average carbon content 0.75 per cent, tungsten 18 per cent, chromium 4 per cent and vanadium 1 per cent.

Explain the various manufacturing processes used in mechanical engineering

Manufacturing Processes

Fig. 1.1. General procedure in Machine Design.

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

1. ***Primary shaping processes.*** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

2. ***Machining processes.*** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbing, etc.

3. ***Surface finishing processes.*** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, superfinishing, sheradizing, etc.

4. ***Joining processes.*** The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties.*** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot peening.

Define Interchangeability

Interchangeability

The term interchangeability is normally employed for the mass production of indentical items within the prescribed limits of sizes. A little consideration will show that in order to maintain the sizes of the part within a close degree of accuracy, a lot of time is required. But even then there will be small variations. If the variations are within certain limits, all parts of equivalent size will be equally fit for operating in machines and mechanisms. Therefore, certain variations are recognised and allowed in the sizes of the mating parts to give the required fitting. This facilitates to select at random from a large number of parts for an assembly and results in a considerable saving in the cost of production. In order to control the size of finished part, with due allowance for error, for interchangeable parts is called ***limit system.***

Define allowance and tolerance

Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) are important from the subject point of view:

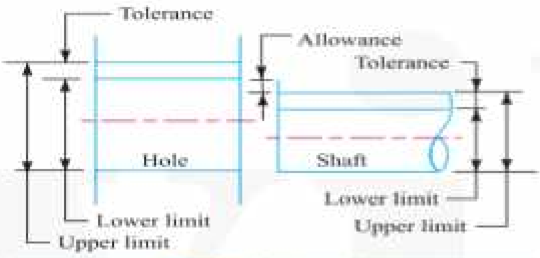


Fig. 3.1. Limits of sizes.

1. ***Nominal size.*** It is the size of a part specified in the drawing as a matter of convenience.

2. ***Basic size.*** It is the size of a part to which all limits of variation (*i.e.* tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

3. ***Actual size.*** It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchangeability of the mating parts.

4. ***Limits of sizes.*** There are two extreme permissible sizes for a dimension of the part as shown in Fig. 3.1. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or ***maximum limit,*** whereas the smallest size of the part is known as ***lower*** or ***minimum limit.***

5. ***Allowance.*** It is the difference between the basic dimensions of the mating parts. The allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative.***

6. ***Tolerance.*** It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be ***unilateral*** or ***bilateral.*** When all the tolerance is allowed on one side of the nominal size, *e.g.* $20^{+0.000}_{-0.004}$, then it is said to be ***unilateral system of tolerance.*** The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit.

(*a*) Unilateral tolerance. (*b*) Bilateral tolerance.

Fig. 3.2. Method of assigning tolerances.

When the tolerance is allowed on both sides of the nominal size, *e.g.* $20^{+0.002}_{-0.002}$, then it is said to be ***bilateral system of tolerance.*** In this case + 0.002 is the upper limit and – 0.002 is the lower limit.

The method of assigning unilateral and bilateral tolerance is shown in Fig. 3.2 (*a*) and (*b*) respectively.

7. ***Tolerance zone.*** It is the zone between the maximum and minimum limit size, as shown in Fig. 3.3.

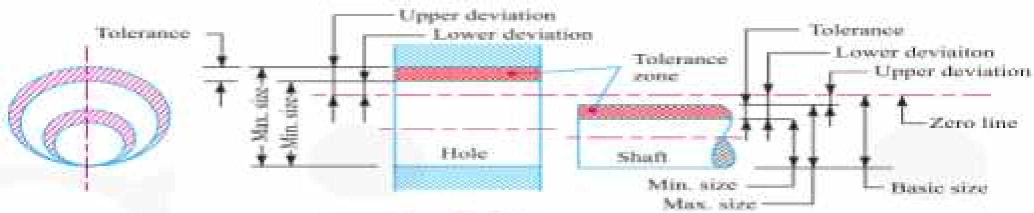


Fig. 3.3. Tolerance zone.

8. ***Zero line.*** It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

9. ***Upper deviation.*** It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol *ES* (Ecart Superior) and of a shaft, it is represented by *es*.

10. ***Lower deviation.*** It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol *EI* (Ecart Inferior) and of a shaft, it is represented by *ei*.

11. ***Actual deviation.*** It is the algebraic difference between an actual size and the corresponding basic size.

12. ***Mean deviation.*** It is the arithmetical mean between the upper and lower deviations.

13. ***Fundamental deviation.*** It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in Fig. 3.4.

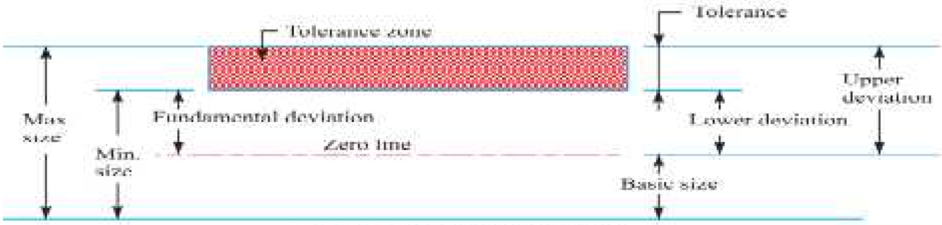


Fig. 3.4. Fundamental deviation.

What is fit? Classify the different types of fits.

Fits

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts. The nature of fit is characterised by the presence and size of clearance and interference.

The ***clearance*** is the amount by which the actual size of the shaft is less than the actual size of the mating hole in an assembly as shown in Fig. 3.5 (*a*). In other words, the clearance is the difference between the sizes of the hole and the shaft before assembly. The difference must be ***positive***.

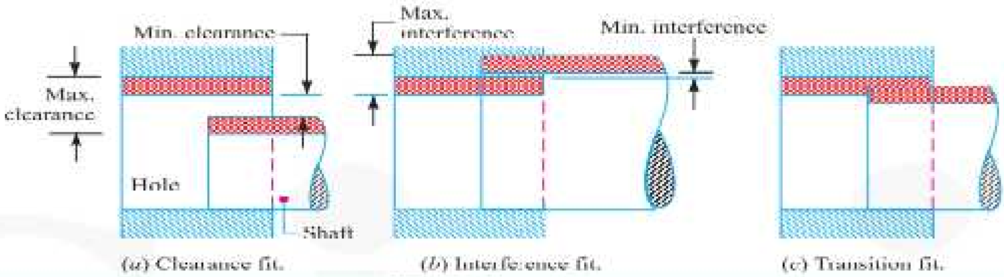


Fig. 3.5. Types of fits.

The ***interference*** is the amount by which the actual size of a shaft is larger than the actual finished size of the mating hole in an assembly as shown in Fig. 3.5 (*b*). In other words, the interference is the arithmetical difference between the sizes of the hole and the shaft, before assembly. The difference must be ***negative***.

1. ***Clearance fit.*** In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig. 3.5 (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft.

The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. ***Interference fit.*** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. 3.5 (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft.

In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference***, whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference***, as shown in Fig. 3.5 (*b*).

The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. ***Transition fit.*** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig. 3.5 (c). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap.

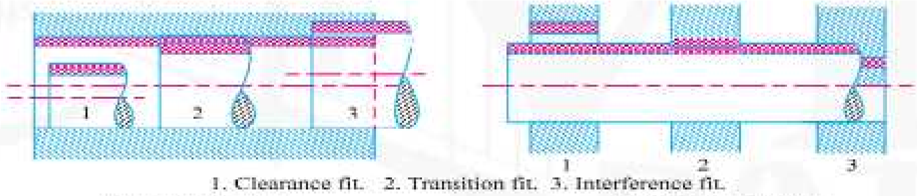
The transition fits may be force fit, tight fit and push fit.

3.16 Basis of Limit System

The following are two bases of limit system:

1. ***Hole basis system.*** When the hole is kept as a constant member (*i.e.* when the lower deviation of the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig. 3.6 (*a*), then the limit system is said to be on a hole basis.

2. ***Shaft basis system.*** When the shaft is kept as a constant member (*i.e.* when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig. 3.6 (*b*), then the limit system is said to be on a shaft basis.

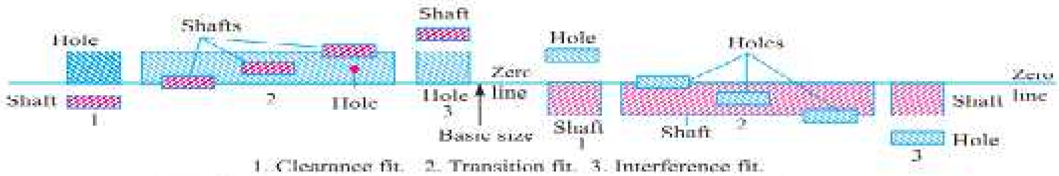


1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. 3.6. Bases of limit system.

The hole basis and shaft basis system may also be shown as in Fig. 3.7, with respect to the zero line.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Fig. 3.7. Bases of limit system.

Discuss various theories of failure.

Theories of Failure Under Static Load

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).
2. Maximum shear stress theory (also known as Guest's or Tresca's theory).
3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).
4. Maximum strain energy theory (also known as Haigh's theory).
5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

Since ductile materials usually fail by yielding *i.e.* when permanent deformations occur in the material and brittle materials fail by fracture, therefore the limiting strength for these two classes of materials is normally measured by different mechanical properties. For ductile materials, the limiting strength is the stress at yield point as determined from simple tension test and it is, assumed to be equal in tension or compression. For brittle materials, the limiting strength is the ultimate stress in tension or compression.

Maximum Principal or Normal Stress Theory (Rankine's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.

Since the limiting strength for ductile materials is yield point stress and for brittle materials (which do not have well defined yield point) the limiting strength is ultimate stress, therefore according to the above theory, taking factor of safety (*F.S.*) into consideration, the maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

where σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

Since the maximum principal or normal stress theory is based on failure in tension or compression and ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials. However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum shear stress in a bi-axial stress system reaches a value equal to the shear stress at yield point in a simple tension test. Mathematically,

$$\tau_{max} = \tau_{yt}/F.S. \quad \ldots(i)$$

where τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation (*i*) may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is mostly used for designing members of ductile materials.

Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the maximum principal (or normal) strain in a bi-axial stress system reaches the limiting value of strain (*i.e.* strain at yield point) as determined from a simple tensile test. The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E}$$

∴ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m \cdot E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \quad ...(i)$$

where σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

From equation (*i*), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure or yielding occurs at a point in a member when the strain energy per unit volume in a bi-axial stress system reaches the limiting strain energy (*i.e.* strain energy at the yield point) per unit volume as determined from simple tension test.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory may be used for ductile materials.

5.14 Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (*i.e.* distortion energy at yield point) per unit volume as determined from a simple tension test. Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Note: The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

Example 5.16. *The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to*

1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory.

Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Solution. Given : P_{t1} = 10 kN ; P_s = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm² ; $1/m$ = 0.3

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. ***According to maximum principal stress theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \quad ...(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4 + 4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

$$\therefore \quad d^2 = 15\,365/100 = 153.65 \quad \text{or} \quad d = 12.4 \text{ mm } \textbf{Ans.}$$

2. According to maximum shear stress theory

We know that maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

$$\therefore \quad d^2 = 9000 / 50 = 180 \quad \text{or} \quad d = 13.42 \text{ mm } \textbf{Ans.}$$

3. According to maximum principal strain theory

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-2.635}{d^2} \text{ kN/mm}^2$$

$$= \frac{-2635}{d^2} \text{ N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7 \text{ mm } \textbf{Ans.}$$

4. According to maximum strain energy theory

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$\therefore \quad d^4 = 26\,724 \quad \text{or} \quad d = 12.78 \text{ mm } \textbf{Ans.}$$

5. According to maximum distortion energy theory

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$\therefore \quad d^4 = 32\,391 \quad \text{or} \quad d = 13.4 \text{ mm } \textbf{Ans.}$$

Example 5.17. *A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN m and a torsional moment 30 kN m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = 210 GPa and poisson's ratio = 0.25.*

Solution. Given : $\sigma_{yt} = 700$ MPa $= 700$ N/mm^2 ; $M = 10$ kN m $= 10 \times 10^6$ N mm ; $T = 30$ kN m $= 30 \times 10^6$ N-mm ; $F.S. = 2$; $E = 210$ GPa $= 210 \times 10^3$ N/mm^2 ; $1/m = 0.25$

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\,T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft (d) by considering the maximum shear stress theory and maximum strain energy theory.

1. ***According to maximum shear stress theory***

We know that maximum shear stress,

$$\tau_{max} = \frac{\sigma_{t1} - \sigma_{t2}}{2} = \frac{1}{2}\left[\frac{211.9 \times 10^6}{d^3} + \frac{110.1 \times 10^6}{d^3}\right] = \frac{161 \times 10^6}{d^3}$$

We also know that according to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{yt}}{2\,F.S.} \quad \text{or} \quad \frac{161 \times 10^6}{d^3} = \frac{700}{2 \times 2} = 175$$

$\therefore$ $d^3 = 161 \times 10^6 / 175 = 920 \times 10^3$ or $d = 97.2$ mm **Ans.**

Note: The value of maximum shear stress (τ_{max}) may also be obtained by using the relation,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2} \times \frac{10^6}{d^3} \times 322 = \frac{161 \times 10^6}{d^3} \text{ N/mm}^2 \qquad \ldots\text{(Same as before)}$$

2. ***According to maximum strain energy theory***

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

or

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9 \times 10^6}{d^3}\right]^2 + \left[\frac{-110.1 \times 10^6}{d^3}\right]^2 - 2 \times \frac{211.9 \times 10^6}{d^3} \times \frac{-110.1 \times 10^6}{d^3} \times 0.25 = \left(\frac{700}{2}\right)^2$$

or

$$\frac{44\,902 \times 10^{12}}{d^6} + \frac{12\,122 \times 10^{12}}{d^6} + \frac{11\,665 \times 10^{12}}{d^6} = 122\,500$$

$$\frac{68\,689 \times 10^{12}}{d^6} = 122\,500$$

$\therefore$ $d^6 = 68\,689 \times 10^{12}/122\,500 = 0.5607 \times 10^{12}$ or $d = 90.8$ mm **Ans.**

Example 5.18. *A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000 N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding.*

Solution. Given: $d = 50$ mm ; $M = 2000$ N m $= 2 \times 10^6$ N mm ; $\sigma_{yt} = 200$ MPa $= 200$ N/mm²

Let T = Maximum torque without causing yielding of the shaft, in N-mm.

1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(50)^3 = 12\,273 \text{ mm}^3$$

$\therefore$ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\,273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\, d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407 \times 10^{-3}\, T \text{ N/mm}^2 \qquad \ldots\left[\because T = \frac{\pi}{16} \times \tau \times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} \text{ N/mm}^2$$

We know that according to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{yt} \qquad \ldots(\text{Taking } F.S. = 1)$$

$\therefore$ $81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} = 200$

$6642.5 + 1.65 + 10^{-9}\,T^2 = (200 - 81.5)^2 = 14\,042$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

or $T = 2118 \times 10^3$ N-mm $= 2118$ N-m **Ans.**

2. *According to maximum shear stress theory*

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$\therefore$ $\sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} = \frac{200}{2} = 100$

$6642.5 + 1.65 \times 10^{-9}\,T^2 = (100)^2 = 10\,000$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$\therefore$ $T = 1426 \times 10^3$ N-mm $= 1426$ N-m **Ans.**

3. *According to maximum distortion strain energy theory*

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65 \times 10^{-9}\,T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65 \times 10^{-9}\,T^2\right] = (200)^2$$

$(81.5)^2 + 3 \times 6642.5 + 3 \times 1.65 \times 10^{-9}\,T^2 = (200)^2$

$26\,570 + 4.95 \times 10^{-9}\,T^2 = 40\,000$

$$T^2 = \frac{40\,000 - 26\,570}{4.95 \times 10^{-9}} = 2713 \times 10^9$$

$\therefore$ $T = 1647 \times 10^3$ N-mm $= 1647$ N-m **Ans.**

Explain preferred numbers and their significance.

Preferred Numbers

When a machine is to be made in several sizes with different powers or capacities, it is necessary to decide what capacities will cover a certain range efficiently with minimum number of sizes. It has been shown by experience that a certain range can be covered efficiently when it follows a geometrical progression with a constant ratio. The preferred numbers are the conventionally rounded off values derived from geometric series including the integral powers of 10 and having as common ratio of the following factors:

$$\sqrt[5]{10},\ \sqrt[10]{10},\ \sqrt[20]{10} \text{ and } \sqrt[40]{10}$$

These ratios are approximately equal to 1.58, 1.26, 1.12 and 1.06. The series of preferred numbers are designated as *R5, R10, R20 and R40 respectively. These four series are called ***basic series***. The other series called ***derived series*** may be obtained by simply multiplying or dividing the basic sizes by 10, 100, etc. The preferred numbers in the series R5 are 1, 1.6, 2.5, 4.0 and 6.3. Table

Define term load, stress and strain, discuss various types of stress- strain

Load

It is defined as **any external force acting upon a machine part.** The following four types of the load are important from the subject point of view:

1. ***Dead or steady load.*** A load is said to be a dead or steady load, when it does not change in magnitude or direction.

2. ***Live or variable load.*** A load is said to be a live or variable load, when it changes continually.

3. ***Suddenly applied or shock loads.*** A load is said to be a suddenly applied or shock load, when it is suddenly applied or removed.

4. ***Impact load.*** A load is said to be an impact load, when it is applied with some initial velocity.

Note: A machine part resists a dead load more easily than a live load and a live load more easily than a shock load.

Stress

When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as ***unit stress*** or simply a ***stress.*** It is denoted by a Greek letter sigma (σ). Mathematically,

Stress, $\sigma = P/A$

where P = Force or load acting on a body, and

A = Cross-sectional area of the body.

In S.I. units, the stress is usually expressed in Pascal (Pa) such that 1 Pa = 1 N/m^2. In actual practice, we use bigger units of stress *i.e.* megapascal (MPa) and gigapascal (GPa), such that

$$1 \text{ MPa} = 1 \times 10^6 \text{ N/m}^2 = 1 \text{ N/mm}^2$$

and $$1 \text{ GPa} = 1 \times 10^9 \text{ N/m}^2 = 1 \text{ kN/mm}^2$$

Strain

When a system of forces or loads act on a body, it undergoes some deformation. This deformation per unit length is known as ***unit strain*** or simply a ***strain.*** It is denoted by a Greek letter epsilon (ε). Mathematically,

Strain, $\varepsilon = \delta l / l$ or $\delta l = \varepsilon . l$

where δl = Change in length of the body, and

l = Original length of the body.

Shear Stress and Strain

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section, as a result of which the body tends to shear off the section, then the stress induced is called ***shear stress.***

Fig. 4.6. Single shearing of a riveted joint.

The corresponding strain is known as ***shear strain*** and it is measured by the angular deformation accompanying the shear stress. The shear stress and shear strain are denoted by the Greek letters tau (τ) and phi (ϕ) respectively. Mathematically,

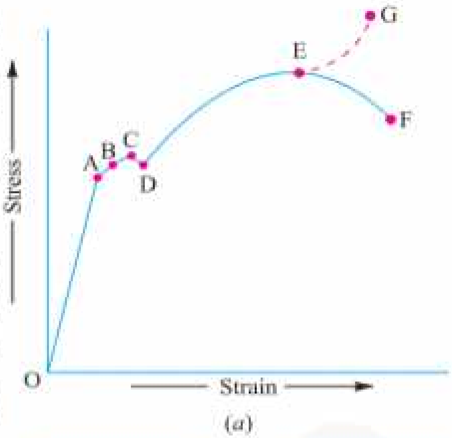
$$\text{Shear stress, } \tau = \frac{\text{Tangential force}}{\text{Resisting area}}$$

Explain stress-strain diagram for mild steel

1. *Proportional limit.* We see from the diagram that from point O to A is a straight line, which represents that the stress is proportional to strain. Beyond point A, the curve slightly deviates from the straight line. It is thus obvious, that Hooke's law holds good up to point A and it is known as ***proportional limit***. It is defined as that stress at which the stress-strain curve begins to deviate from the straight line.

2. *Elastic limit.* It may be noted that even if the load is increased beyond point A upto the point B, the material will regain its shape and size when the load is removed. This means that the material has elastic properties up to the point B. This point is known as ***elastic limit.*** It is defined as the stress developed in the material without any permanent set.

Note: Since the above two limits are very close to each other, therefore, for all practical purposes these are taken to be equal.



(*a*)


(*b*) Shape of specimen after elongation.

Fig. 4.12. Stress-strain diagram for a mild steel.

3. *Yield point.* If the material is stressed beyond point B, the plastic stage will reach *i.e.* on the removal of the load, the material will not be able to recover its original size and shape. A little consideration will show that beyond point B, the strain increases at a faster rate with any increase in the stress until the point C is reached. At this point, the material yields before the load and there is an appreciable strain without any increase in stress. In case of mild steel, it will be seen that a small load drops to D, immediately after yielding commences. Hence there are two yield points C and D. The points C and D are called the ***upper*** and ***lower yield points*** respectively. The stress corresponding to yield point is known as ***yield point stress.***

4. *Ultimate stress.* At D, the specimen regains some strength and higher values of stresses are required for higher strains, than those between A and D. The stress (or load) goes on increasing till the point E is reached. The gradual increase in the strain (or length) of the specimen is followed with the uniform reduction of its cross-sectional area. The work done, during stretching the specimen, is transformed largely into heat and the specimen becomes hot. At E, the stress, which attains its maximum value is known as ***ultimate stress.*** It is defined as the largest stress obtained by dividing the largest value of the load reached in a test to the original cross-sectional area of the test piece.

5. *Breaking stress.* After the specimen has reached the ultimate stress, a neck is formed, which decreases the cross-sectional area of the specimen, as shown in Fig. 4.12 (*b*). A little consideration will show that the stress (or load) necessary to break away the specimen, is less than the maximum stress. The stress is, therefore, reduced until the specimen breaks away at point F. The stress corresponding to point F is known as ***breaking stress.***

6. *Percentage reduction in area.* It is the difference between the original cross-sectional area and cross-sectional area at the neck (*i.e.* where the fracture takes place). This difference is expressed as percentage of the original cross-sectional area.

Let A = Original cross-sectional area, and

a = Cross-sectional area at the neck.

Then reduction in area $= A - a$

and percentage reduction in area $= \dfrac{A - a}{A} \times 100$

7. *Percentage elongation.* It is the percentage increase in the standard gauge length (*i.e.* original length) obtained by measuring the fractured specimen after bringing the broken parts together.

Let l = Gauge length or original length, and

L = Length of specimen after fracture or final length.

$\therefore$ Elongation $= L - l$

and percentage elongation $= \dfrac{L - l}{l} \times 100$

Define working stress and explain the term factor of safety

Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the ***working stress*** or ***design stress.*** It is also known as ***safe*** or ***allowable stress.***

Factor of Safety

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\therefore \quad \text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

This relation may also be used for ductile materials.

Note: The above relations for factor of safety are for static loading.

Example 4.15. *A copper bar 50 mm in diameter is placed within a steel tube 75 mm external diameter and 50 mm internal diameter of exactly the same length. The two pieces are rigidly fixed together by two pins 18 mm in diameter, one at each end passing through the bar and tube. Calculate the stress induced in the copper bar, steel tube and pins if the temperature of the combination is raised by 50°C. Take E_s = 210 GN/m² ; E_c = 105 GN/m² ; $\alpha_s = 11.5 \times 10^{-6}$/°C and $\alpha_c = 17 \times 10^{-6}$/°C.*

Solution. Given: d_c = 50 mm ; d_{se} = 75 mm ; d_{si} = 50 mm ; d_p = 18 mm = 0.018 m ; t = 50°C; E_s = 210 GN/m² = 210 × 10⁹ N/m² ; E_c = 105 GN/m² = 105 × 10⁹ N/m² ; $\alpha_s = 11.5 \times 10^{-6}$/°C ; $\alpha_c = 17 \times 10^{-6}$/°C

The copper bar in a steel tube is shown in Fig. 4.18.

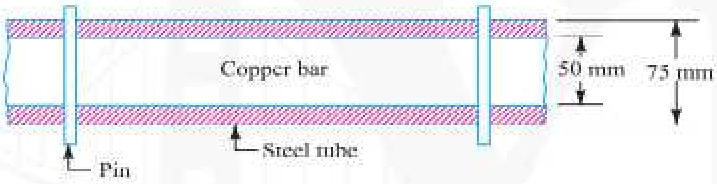


Fig. 4.18

We know that cross-sectional area of the copper bar,

$$A_c = \frac{\pi}{4}(d_c)^2 = \frac{\pi}{4}(50)^2 = 1964 \text{ mm}^2 = 1964 \times 10^{-6} \text{ m}^2$$

and cross-sectional area of the steel tube,

$$A_s = \frac{\pi}{4}\left[(d_{se})^2 - (d_{si})^2\right] = \frac{\pi}{4}\left[(75)^2 - (50)^2\right] = 2455 \text{ mm}^2$$
$$= 2455 \times 10^{-6} \text{ m}^2$$

Let $\quad l$ = Length of the copper bar and steel tube.

We know that free expansion of copper bar

$$= \alpha_c \,.\, l \,.\, t = 17 \times 10^{-6} \times l \times 50 = 850 \times 10^{-6}\, l$$

and free expansion of steel tube

$$= \alpha_s \,.\, l \,.\, t = 11.5 \times 10^{-6} \times l \times 50 = 575 \times 10^{-6}\, l$$

∴ Difference in free expansion

$$= 850 \times 10^{-6}\, l - 575 \times 10^{-6}\, l = 275 \times 10^{-6}\, l \qquad ...(i)$$

$\therefore$ Reduction in length of copper bar due to force P $= \frac{P.l}{A_c . E_c}$

$$= \frac{P.l}{1964 \times 10^{-6} \times 105 \times 10^{9}} = \frac{P.l}{206.22 \times 10^{6}} \text{ m}$$

and increase in length of steel bar due to force P

$$= \frac{P.l}{A_s . E_s} = \frac{P.l}{2455 \times 10^{-6} \times 210 \times 10^{9}} = \frac{P.l}{515.55 \times 10^{6}} \text{ m}$$

$$\therefore \text{Net effect in length} = \frac{P.l}{206.22 \times 10^{6}} + \frac{P.l}{515.55 \times 10^{6}}$$

$$= 4.85 \times 10^{-9} P.l + 1.94 \times 10^{-9} P.l = 6.79 \times 10^{-9} P.l$$

Equating this net effect in length to the difference in free expansion, we have

$$6.79 \times 10^{-9} P.l = 275 \times 10^{-6} l \quad \text{or} \quad P = 40\,500 \text{ N}$$

Stress induced in the copper bar, steel tube and pins

We know that stress induced in the copper bar,

$$\sigma_c = P / A_c = 40\,500 / (1964 \times 10^{-6}) = 20.62 \times 10^{6} \text{ N/m}^2 = 20.62 \text{ MPa} \textbf{ Ans.}$$

Stress induced in the steel tube,

Stress induced in the steel tube,

$$\sigma_s = P / A_s = 40\,500 / (2455 \times 10^{-6}) = 16.5 \times 10^{6} \text{ N / m}^2 = 16.5 \text{ MPa} \textbf{ Ans.}$$

and shear stress induced in the pins,

$$\tau_p = \frac{P}{2\,A_p} = \frac{40\,500}{2 \times \frac{\pi}{4}(0.018)^2} = 79.57 \times 10^{6} \text{ N/m}^2 = 79.57 \text{ MPa} \textbf{ Ans.}$$

...($\because$ The pin is in double shear)

Define Poisson's Ratio, Volumetric strain, Bulk modulus, Resilience, lateral and linear strain

Linear and Lateral Strain

Consider a circular bar of diameter d and length l, subjected to a tensile force P as shown in Fig. 4.19 (a).

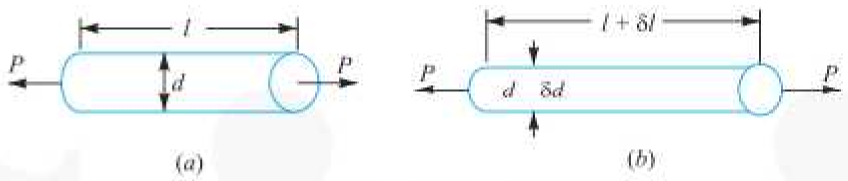


Fig. 4.19. Linear and lateral strain.

A little consideration will show that due to tensile force, the length of the bar increases by an amount δl and the diameter decreases by an amount δd, as shown in Fig. 4.19 (b). Similarly, if the bar is subjected to a compressive force, the length of bar will decrease which will be followed by increase in diameter.

It is thus obvious, that every direct stress is accompanied by a strain in its own direction which is known as ***linear strain*** and an opposite kind of strain in every direction, at right angles to it, is known as ***lateral strain.***

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Volumetric Strain

When a body is subjected to a system of forces, it undergoes some changes in its dimensions. In other words, the volume of the body is changed. The ratio of the change in volume to the original volume is known as ***volumetric strain.*** Mathematically, volumetric strain,

$$\varepsilon_v = \delta V / V$$

where δV = Change in volume, and V = Original volume.

Notes : 1. Volumetric strain of a rectangular body subjected to an axial force is given as

$$\varepsilon_v = \frac{\delta v}{V} = \varepsilon\left(1 - \frac{2}{m}\right); \text{ where } \varepsilon = \text{Linear strain.}$$

2. Volumetric strain of a rectangular body subjected to three mutually perpendicular forces is given by

$$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

where ε_x, ε_y and ε_z are the strains in the directions x-axis, y-axis and z-axis respectively.

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as ***bulk modulus.*** It is usually denoted by K. Mathematically, bulk modulus,

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$

Relation Between Bulk Modulus and Young's Modulus

The bulk modulus (K) and Young's modulus (E) are related by the following relation,

$$K = \frac{m.E}{3\,(m-2)} = \frac{E}{3\,(1-2\mu)}$$

Relation Between Young's Modulus and Modulus of Rigidity

The Young's modulus (E) and modulus of rigidity (G) are related by the following relation,

$$G = \frac{m.E}{2\,(m+1)} = \frac{E}{2\,(1+\mu)}$$

Example 4.18. *An unknown weight falls through 10 mm on a collar rigidly attached to the lower end of a vertical bar 3 m long and 600 mm² in section. If the maximum instantaneous extension is known to be 2 mm, what is the corresponding stress and the value of unknown weight? Take E = 200 kN/mm².*

Solution. Given : h = 10 mm ; l = 3 m = 3000 mm ; A = 600 mm² ; δl = 2 mm ; E = 200 kN/mm² = 200 × 10³ N/mm²

Stress in the bar

Let σ = Stress in the bar.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma . l}{\delta l}$$

Value of the unknown weight

Let W = Value of the unknown weight.

We know that

$$\sigma = \frac{W}{A}\left[1 + \sqrt{1 + \frac{2h\,A\,E}{Wl}}\right]$$

$$\frac{400}{3} = \frac{W}{600}\left[1 + \sqrt{1 + \frac{2 \times 10 \times 600 \times 200 \times 10^3}{W \times 3000}}\right]$$

$$\frac{400 \times 600}{3\,W} = 1 + \sqrt{1 + \frac{800\,000}{W}}$$

$$\frac{80\,000}{W} - 1 = \sqrt{1 + \frac{800\,000}{W}}$$

Squaring both sides,

$$\frac{6400 \times 10^6}{W^2} + 1 - \frac{160\,000}{W} = 1 + \frac{800\,000}{W}$$

$$\frac{6400 \times 10^2}{W} - 16 = 80 \quad \text{or} \quad \frac{6400 \times 10^2}{W} = 96$$

$\therefore$ $W = 6400 \times 10^2 / 96 = 6666.7$ N **Ans.**

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy.*** The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called

Resilience

When a body is loaded within elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy in itself. On removing the load, the energy stored is given off as in the case of a spring. This energy, which is absorbed in a body when strained within elastic limit, is known as ***strain energy***. The strain energy is always capable of doing some work.

The strain energy stored in a body due to external loading, within elastic limit, is known as ***resilience*** and the maximum energy which can be stored in a body up to the elastic limit is called ***proof resilience***. The proof resilience per unit volume of a material is known as ***modulus of resilience***. It is an important property of a material and gives capacity of the material to bear impact or

Define torsional shear stress, torsional rigidity and derive the torsion equation

Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion***. The stress set up by torsion is known as ***torsional shear stress***. It is zero at the centroidal axis and maximum at the outer surface.

Consider a shaft fixed at one end and subjected to a torque (T) at the other end as shown in Fig. 5.1. As a result of this torque, every cross section of the shaft is subjected to torsional shear stress. We have discussed above that the

torsional shear stress is zero at the centroidal axis and maximum at the outer surface. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \ldots(i)$$

where
τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,
r = Radius of the shaft,
T = Torque or twisting moment,
J = Second moment of area of the section about its polar axis or polar moment of inertia,
C = Modulus of rigidity for the shaft material,
l = Length of the shaft, and
θ = Angle of twist in radians on a length l.

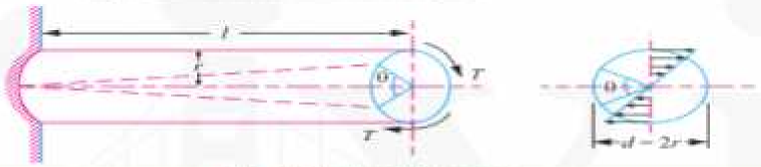


Fig. 5.1. Torsional shear stress.

The equation (i) is known as ***torsion equation***. It is based on the following assumptions:

1. The material of the shaft is uniform throughout.
2. The twist along the length of the shaft is uniform.
3. The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.
4. All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.
5. The maximum shear stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

Notes : 1. Since the torsional shear stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the torsional shear stress at a distance x from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

2. From equation (i), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (d), the polar moment of inertia,

$$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

$$\therefore \quad T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter (d_o) and internal diameter (d_i), the polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \text{ and } r = \frac{d_o}{2}$$

$$\therefore \quad T = \tau \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$

$$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \ldots\left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$

3. The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

4. The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2\pi N . T}{60} = T.\omega \qquad \ldots\left(\because \omega = \frac{2\pi N}{60}\right)$$

where T = Torque transmitted in N-m, and

ω = Angular speed in rad/s.

Example 5.1. *A shaft is transmitting 100 kW at 160 r.p.m. Find a suitable diameter for the shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$; $\tau = 70$ MPa $= 70$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted (P),

$$100 \times 10^3 = \frac{2\pi N . T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque (T_{max}),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\ d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm} \quad \textbf{Ans.}$$

Example 5.2. *A steel shaft 35 mm in diameter and 1.2 m long held rigidly at one end has a hand wheel 500 mm in diameter keyed to the other end. The modulus of rigidity of steel is 80 GPa.*

1. What load applied to tangent to the rim of the wheel produce a torsional shear of 60 MPa?

2. How many degrees will the wheel turn when this load is applied?

Solution. Given : $d = 35$ mm or $r = 17.5$ mm ; $l = 1.2$ m $= 1200$ mm ; $D = 500$ mm or $R = 250$ mm ; $C = 80$ GPa $= 80$ kN/mm$^2 = 80 \times 10^3$ N/mm^2 ; $\tau = 60$ MPa $= 60$ N/mm^2

1. Load applied to the tangent to the rim of the wheel

Let W = Load applied (in newton) to tangent to the rim of the wheel.

We know that torque applied to the hand wheel,

$$T = W.R = W \times 250 = 250\ W \text{ N-mm}$$

and polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = \frac{\pi}{32}(35)^4 = 147.34 \times 10^3 \text{ mm}^4$$

We know that $\quad \dfrac{T}{J} = \dfrac{\tau}{r}$

$$\therefore \quad \frac{250\ W}{147.34 \times 10^3} = \frac{60}{17.5} \quad \text{or} \quad W = \frac{60 \times 147.34 \times 10^3}{17.5 \times 250} = 2020 \text{ N} \quad \textbf{Ans.}$$

2. Number of degrees which the wheel will turn when load W = 2020 N is applied

Let θ = Required number of degrees.

We know that $\quad \dfrac{T}{J} = \dfrac{C.\theta}{l}$

$$\therefore \quad \theta = \frac{T.l}{C.J} = \frac{250 \times 2020 \times 1200}{80 \times 10^3 \times 147.34 \times 10^3} = 0.05^\circ \quad \textbf{Ans.}$$

Example 5.3. *A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.*

Solution. Given : $P = 97.5$ kW $= 97.5 \times 10^3$ W ; $N = 180$ r.p.m. ; $\tau = 60$ MPa $= 60$ N/mm^2 ; $\theta = 1^\circ = \pi/180 = 0.0174$ rad ; $l = 3$ m $= 3000$ mm ; $C = 80$ GPa $= 80 \times 10^9$ N/m$^2 = 80 \times 10^3$ N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

We know that the power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\pi N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\ T$$

$$\therefore \quad T = 97.5 \times 10^3/18.852 = 5172 \text{ N-m} = 5172 \times 10^3 \text{ N-mm}$$

Now let us find the diameter of the shaft based on the strength and stiffness.

1. *Considering strength of the shaft*

We know that the torque transmitted (T),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\ d^3$$

$\therefore$ $d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3$ or $d = 76$ mm ...(*i*)

2. *Considering stiffness of the shaft*

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\ d^4$$

We know that $\frac{T}{J} = \frac{C.\theta}{l}$

$$\frac{5172 \times 10^3}{0.0982\ d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \text{ or } \frac{52.7 \times 10^6}{d^4} = 0.464$$

$\therefore$ $d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6$ or $d = 103$ mm ...(*ii*)

Taking larger of the two values, we shall provide $d = 103$ say 105 mm **Ans.**

Example 5.4. *A hollow shaft is required to transmit 600 kW at 110 r.p.m., the maximum torque being 20% greater than the mean. The shear stress is not to exceed 63 MPa and twist in a length of 3 metres not to exceed 1.4 degrees. Find the external diameter of the shaft, if the internal diameter to the external diameter is 3/8. Take modulus of rigidity as 84 GPa.*

Solution. Given : $P = 600$ kW $= 600 \times 10^3$ W ; $N = 110$ r.p.m. ; $T_{max} = 1.2\ T_{mean}$; $\tau = 63$ MPa $= 63$ N/mm^2 ; $l = 3$ m $= 3000$ mm ; $\theta = 1.4 \times \pi / 180 = 0.024$ rad ; $k = d_i / d_o = 3/8$; $C = 84$ GPa $= 84 \times 10^9$ N/m$^2 = 84 \times 10^3$ N/mm^2

Let T_{mean} = Mean torque transmitted by the shaft,

d_o = External diameter of the shaft, and

d_i = Internal diameter of the shaft.

We know that power transmitted by the shaft (P),

$$600 \times 10^3 = \frac{2\pi N.T_{mean}}{60} = \frac{2\pi \times 110 \times T_{mean}}{60} = 11.52\ T_{mean}$$

$\therefore$ $T_{mean} = 600 \times 10^3/11.52 = 52 \times 10^3$ N-m $= 52 \times 10^6$ N-mm

and maximum torque transmitted by the shaft,

$$T_{max} = 1.2\ T_{mean} = 1.2 \times 52 \times 10^6 = 62.4 \times 10^6 \text{ N-mm}$$

Now let us find the diameter of the shaft considering strength and stiffness.

1. *Considering strength of the shaft*

We know that maximum torque transmitted by the shaft,

$$T_{max} = \frac{\pi}{16} \times \tau\ (d_o)^3\ (1 - k^4)$$

$$62.4 \times 10^6 = \frac{\pi}{16} \times 63 \times (d_o)^3 \left[1 - \left(\frac{3}{8}\right)^4\right] = 12.12\ (d_o)^3$$

$\therefore$ $(d_o)^3 = 62.4 \times 10^6/12.12 = 5.15 \times 10^6$ or $d_o = 172.7$ mm ...(*i*)

2. *Considering stiffness of the shaft*

We know that polar moment of inertia of a hollow circular section,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{32}(d_o)^4 \left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$= \frac{\pi}{32}(d_o)^4\ (1 - k^4) = \frac{\pi}{32}(d_o)^4 \left[1 - \left(\frac{3}{8}\right)^4\right] = 0.0962\ (d_o)^4$$

We also know that

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{62.4 \times 10^6}{0.0962\ (d_o)^4} = \frac{84 \times 10^3 \times 0.024}{3000} \text{ or } \frac{648.6 \times 10^6}{(d_o)^4} = 0.672$$

$\therefore$ $(d_o)^4 = 648.6 \times 10^6/0.672 = 964 \times 10^6$ or $d_o = 176.2$ mm ...(*ii*)

Taking larger of the two values, we shall provide

$d_o = 176.2$ say 180 mm **Ans.**

Example 4.1. *A coil chain of a crane required to carry a maximum load of 50 kN, is shown in Fig. 4.3.*

Fig. 4.3

Find the diameter of the link stock, if the permissible tensile stress in the link material is not to exceed 75 MPa.

Solution. Given : $P = 50 \text{ kN} = 50 \times 10^3$ N ; $\sigma_t = 75$ MPa $= 75$ N/mm²

Let d = Diameter of the link stock in mm.

$\therefore$ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$

We know that the maximum load (P),

$$50 \times 10^3 = \sigma_t \cdot A = 75 \times 0.7854\, d^2 = 58.9\, d^2$$

$\therefore$ $d^2 = 50 \times 10^3 / 58.9 = 850$ or $d = 29.13$ say 30 mm **Ans.**

Example 4.2. *A cast iron link, as shown in Fig. 4.4, is required to transmit a steady tensile load of 45 kN. Find the tensile stress induced in the link material at sections A-A and B-B.*

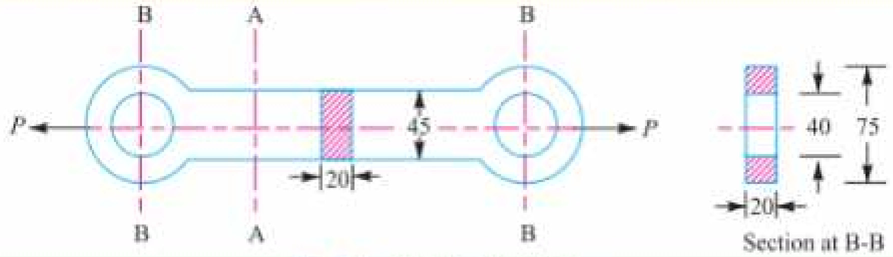


Fig. 4.4. All dimensions in mm.

Solution. Given : $P = 45 \text{ kN} = 45 \times 10^3$ N

Tensile stress induced at section A-A

We know that the cross-sectional area of link at section A-A,

$$A_1 = 45 \times 20 = 900 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section A-A,

$$\sigma_{t1} = \frac{P}{A_1} = \frac{45 \times 10^3}{900} = 50 \text{ N/mm}^2 = 50 \text{ MPa} \quad \textbf{Ans.}$$

Tensile stress induced at section B-B

We know that the cross-sectional area of link at section B-B,

$$A_2 = 20\,(75 - 40) = 700 \text{ mm}^2$$

$\therefore$ Tensile stress induced at section B-B,

$$\sigma_{t2} = \frac{P}{A_2} = \frac{45 \times 10^3}{700} = 64.3 \text{ N/mm}^2 = 64.3 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.3. *A hydraulic press exerts a total load of 3.5 MN. This load is carried by two steel rods, supporting the upper head of the press. If the safe stress is 85 MPa and E = 210 kN/mm², find : 1. diameter of the rods, and 2. extension in each rod in a length of 2.5 m.*

Solution. Given : $P = 3.5 \text{ MN} = 3.5 \times 10^6$ N ; $\sigma_t = 85$ MPa $= 85$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm² ; $l = 2.5 \text{ m} = 2.5 \times 10^3$ mm

1. Diameter of the rods

Let d = Diameter of the rods in mm.

$\therefore$ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$

Since the load P is carried by two rods, therefore load carried by each rod,

$$P_1 = \frac{P}{2} = \frac{3.5 \times 10^6}{2} = 1.75 \times 10^6 \text{ N}$$

We know that load carried by each rod (P_1),

$$1.75 \times 10^6 = \sigma_t \cdot A = 85 \times 0.7854\, d^2 = 66.76\, d^2$$

$\therefore$ $d^2 = 1.75 \times 10^6 / 66.76 = 26\,213$ or $d = 162$ mm **Ans.**

2. Extension in each rod

Let δl = Extension in each rod.

We know that Young's modulus (E),

$$210 \times 10^3 = \frac{P_1 \times l}{A \times \delta l} = \frac{\sigma_t \times l}{\delta l} = \frac{85 \times 2.5 \times 10^3}{\delta l} = \frac{212.5 \times 10^3}{\delta l} \quad \ldots \left(\because \frac{P_1}{A} = \sigma_t\right)$$

$\therefore$ $\delta l = 212.5 \times 10^3 / (210 \times 10^3) = 1.012$ mm **Ans.**

Example 4.4. *A rectangular base plate is fixed at each of its four corners by a 20 mm diameter bolt and nut as shown in Fig. 4.5. The plate rests on washers of 22 mm internal diameter and 50 mm external diameter. Copper washers which are placed between the nut and the plate are of 22 mm internal diameter and 44 mm external diameter.*

If the base plate carries a load of 120 kN (including self-weight, which is equally distributed on the four corners), calculate the stress on the lower washers before the nuts are tightened.

What could be the stress in the upper and lower washers, when the nuts are tightened so as to produce a tension of 5 kN on each bolt?

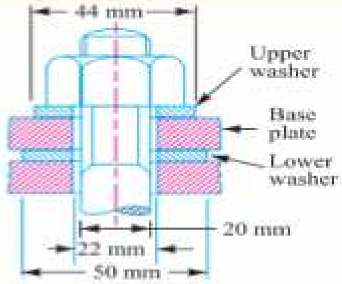


Fig. 4.5

Solution. Given : $d = 20$ mm ; $d_1 = 22$ mm ; $d_2 = 50$ mm ; $d_3 = 22$ mm ; $d_4 = 44$ mm ; $P_1 = 120$ kN ; $P_2 = 5$ kN

Stress on the lower washers before the nuts are tightened

We know that area of lower washers,

$$A_1 = \frac{\pi}{4}\left[(d_2)^2 - (d_1)^2\right] = \frac{\pi}{4}\left[(50)^2 - (22)^2\right] = 1583 \text{ mm}^2$$

and area of upper washers,

$$A_2 = \frac{\pi}{4}\left[(d_4)^2 - (d_3)^2\right] = \frac{\pi}{4}\left[(44)^2 - (22)^2\right] = 1140 \text{ mm}^2$$

Since the load of 120 kN on the four washers is equally distributed, therefore load on each lower washer before the nuts are tightened,

$$P_1 = \frac{120}{4} = 30 \text{ kN} = 30\,000 \text{ N}$$

We know that stress on the lower washers before the nuts are tightened,

$$\sigma_{c1} = \frac{P_1}{A_1} = \frac{30\,000}{1583} = 18.95 \text{ N/mm}^2 = 18.95 \text{ MPa} \quad \textbf{Ans.}$$

Stress on the upper washers when the nuts are tightened

Tension on each bolt when the nut is tightened,

$$P_2 = 5 \text{ kN} = 5000 \text{ N}$$

$\therefore$ Stress on the upper washers when the nut is tightened,

$$\sigma_{c2} = \frac{P_2}{A_2} = \frac{5000}{1140} = 4.38 \text{ N/mm}^2 = 4.38 \text{ MPa} \quad \textbf{Ans.}$$

Stress on the lower washers when the nuts are tightened

We know that the stress on the lower washers when the nuts are tightened,

$$\sigma_{c3} = \frac{P_1 + P_2}{A_1} = \frac{30\,000 + 5000}{1583} = 22.11 \text{ N/mm}^2 = 22.11 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.5. *The piston rod of a steam engine is 50 mm in diameter and 600 mm long. The diameter of the piston is 400 mm and the maximum steam pressure is 0.9 N/mm². Find the compression of the piston rod if the Young's modulus for the material of the piston rod is 210 kN/mm².*

Solution. Given : $d = 50$ mm ; $l = 600$ mm ; $D = 400$ mm ; $p = 0.9$ N/mm² ; $E = 210$ kN/mm² $= 210 \times 10^3$ N/mm²

Let δl = Compression of the piston rod.

We know that cross-sectional area of piston,

$$= \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(400)^2 = 125\,680 \text{ mm}^2$$

$\therefore$ Maximum load acting on the piston due to steam,

$$P = \text{Cross-sectional area of piston} \times \text{Steam pressure}$$

$$= 125\,680 \times 0.9 = 113\,110 \text{ N}$$

We also know that cross-sectional area of piston rod,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(50)^2$$

$$= 1964 \text{ mm}^2$$

and Young's modulus (E),

$$210 \times 10^3 = \frac{P \times l}{A \times \delta l}$$

$$= \frac{113\,110 \times 600}{1964 \times \delta l} = \frac{34\,555}{\delta l}$$

$$\therefore \quad \delta l = 34\,555 / (210 \times 10^3)$$

$$= 0.165 \text{ mm} \quad \textbf{Ans.}$$

Example 4.6. *Calculate the force required to punch a circular blank of 60 mm diameter in a plate of 5 mm thick. The ultimate shear stress of the plate is 350 N/mm².*

Solution. Given: $d = 60$ mm ; $t = 5$ mm ; $\tau_u = 350$ N/mm²

We know that area under shear,

$$A = \pi\, d \times t = \pi \times 60 \times 5 = 942.6 \text{ mm}^2$$

and force required to punch a hole,

$$P = A \times \tau_u = 942.6 \times 350 = 329\ 910 \text{ N} = 329.91 \text{ kN} \quad \textbf{Ans.}$$

Example 4.7. *A pull of 80 kN is transmitted from a bar X to the bar Y through a pin as shown in Fig. 4.8.*

If the maximum permissible tensile stress in the bars is 100 N/mm² and the permissible shear stress in the pin is 80 N/mm², find the diameter of bars and of the pin.

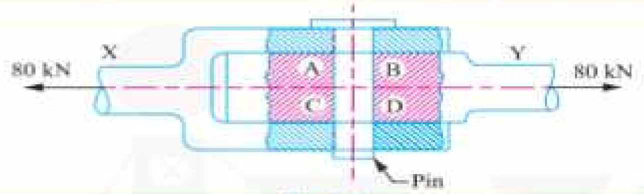


Fig. 4.8

Solution. Given : $P = 80$ kN $= 80 \times 10^3$ N;
$\sigma_t = 100$ N/mm² ; $\tau = 80$ N/mm²

Diameter of the bars

Let D_b = Diameter of the bars in mm.

$\therefore$ Area, $A_b = \dfrac{\pi}{4}\,(D_b)^2 = 0.7854\,(D_b)^2$

We know that permissible tensile stress in the bar (σ_t),

$$100 = \frac{P}{A_b} = \frac{80 \times 10^3}{0.7854\,(D_b)^2} = \frac{101\ 846}{(D_b)^2}$$

$\therefore \quad (D_b)^2 = 101\ 846 / 100 = 1018.46$

or $\quad D_b = 32$ mm **Ans.**

Diameter of the pin

Let D_p = Diameter of the pin in mm.

Since the tensile load P tends to shear off the pin at two sections *i.e.* at AB and CD, therefore the pin is in double shear.

$\therefore$ Resisting area,

$$A_p = 2 \times \frac{\pi}{4}\,(D_p)^2 = 1.571\,(D_p)^2$$

We know that permissible shear stress in the pin (τ),

$$80 = \frac{P}{A_p} = \frac{80 \times 10^3}{1.571\,(D_p)^2} = \frac{50.9 \times 10^3}{(D_p)^2}$$

$\therefore \quad (D_p)^2 = 50.9 \times 10^3/80 = 636.5$ or $D_p = 25.2$ mm **Ans.**

Example 4.8. *Two plates 16 mm thick are joined by a double riveted lap joint as shown in Fig. 4.11. The rivets are 25 mm in diameter.*

Find the crushing stress induced between the plates and the rivet, if the maximum tensile load on the joint is 48 kN.

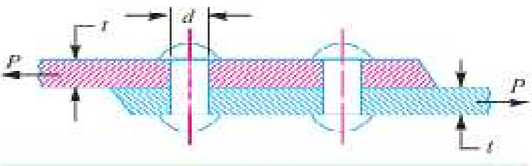


Fig. 4.11

Solution. Given : $t = 16$ mm ; $d = 25$ mm ;
$P = 48$ kN $= 48 \times 10^3$ N

Since the joint is double riveted, therefore, strength of two rivets in bearing (or crushing) is taken. We know that crushing stress induced between the plates and the rivets,

$$\sigma_c = \frac{P}{d.t.n} = \frac{48 \times 10^3}{25 \times 16 \times 2} = 60 \text{ N/mm}^2 \quad \textbf{Ans.}$$

Example 4.9. *A journal 25 mm in diameter supported in sliding bearings has a maximum end reaction of 2500 N. Assuming an allowable bearing pressure of 5 N/mm², find the length of the sliding bearing.*

Solution. Given : $d = 25$ mm ; $P = 2500$ N ; $p_b = 5$ N/mm²

Let l = Length of the sliding bearing in mm.

We know that the projected area of the bearing,

$$A = l \times d = l \times 25 = 25\, l \text{ mm}^2$$

$\therefore$ Bearing pressure (p_b),

$$5 = \frac{P}{A} = \frac{2500}{25\, l} = \frac{100}{l} \quad \text{or} \quad l = \frac{100}{5} = 20 \text{ mm} \quad \textbf{Ans.}$$

Example 4.10. *A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded :*

Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN.

Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation.

Solution. Given : $D = 12$ mm ; $l = 60$ mm ; $L = 80$ mm ; $d = 7$ mm ; $W_y = 3.4$ kN $= 3400$ N; $W_u = 6.1$ kN $= 6100$ N

We know that original area of the rod,

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$$

and final area of the rod,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(7)^2 = 38.5 \text{ mm}^2$$

1. *Yield stress*

We know that yield stress

$$= \frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 = 30.1 \text{ MPa} \quad \textbf{Ans.}$$

2. *Ultimate tensile stress*

We know the ultimate tensile stress

$$= \frac{W_u}{A} = \frac{6100}{113} = 54 \text{ N/mm}^2 = 54 \text{ MPa} \quad \textbf{Ans.}$$

3. *Percentage reduction in area*

We know that percentage reduction in area

$$= \frac{A - a}{A} = \frac{113 - 38.5}{113} = 0.66 \text{ or } 66\% \quad \textbf{Ans.}$$

4. *Percentage elongation*

We know that percentage elongation

$$= \frac{L - l}{L} = \frac{80 - 60}{80} = 0.25 \text{ or } 25\% \quad \textbf{Ans.}$$

Example 4.11. *A bar 3 m long is made of two bars, one of copper having $E = 105$ GN/m² and the other of steel having $E = 210$ GN/m². Each bar is 25 mm broad and 12.5 mm thick. This compound bar is stretched by a load of 50 kN. Find the increase in length of the compound bar and the stress produced in the steel and copper. The length of copper as well as of steel bar is 3 m each.*

Solution. Given : $l_c = l_s = 3$ m $= 3 \times 10^3$ mm ; $E_c = 105$ GN/m² $= 105$ kN/mm² ; $E_s = 210$ GN/m² $= 210$ kN/mm² ; $b = 25$ mm ; $t = 12.5$ mm ; $P = 50$ kN

Increase in length of the compound bar

Let δl = Increase in length of the compound bar.

The compound bar is shown in Fig. 4.14. We know that cross-sectional area of each bar,

$$A_c = A_s = b \times t = 25 \times 12.5 = 312.5 \text{ mm}^2$$

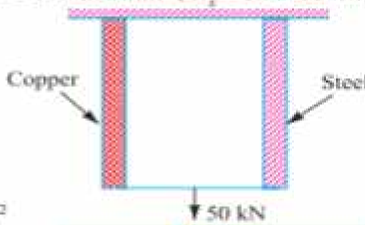


Fig. 4.14

$\therefore$ Load shared by the copper bar,

$$P_c = P \times \frac{A_c . E_c}{A_c . E_c + A_s . E_s} = P \times \frac{E_c}{E_c + E_s} \quad \ldots (\because A_c = A_s)$$

$$= 50 \times \frac{105}{105 + 210} = 16.67 \text{ kN}$$

and load shared by the steel bar,

$$P_s = P - P_c = 50 - 16.67 = 33.33 \text{ kN}$$

Since the elongation of both the bars is equal, therefore

$$\delta l = \frac{P_c . l_c}{A_c . E_c} = \frac{P_s . l_s}{A_s . E_s} = \frac{16.67 \times 3 \times 10^3}{312.5 \times 105} = 1.52 \text{ mm} \quad \textbf{Ans.}$$

Stress produced in the steel and copper bar

We know that stress produced in the steel bar,

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{210}{105} \times \sigma_c = 2\,\sigma_c$$

and total load, $P = P_s + P_c = \sigma_s A_s + \sigma_c A_c$

$\therefore$ $50 = 2\,\sigma_c \times 312.5 + \sigma_c \times 312.5 = 937.5\,\sigma_c$

or $\sigma_c = 50 / 937.5 = 0.053$ kN/mm² $= 53$ N/mm² $= 53$ MPa **Ans.**

and $\sigma_s = 2\,\sigma_c = 2 \times 53 = 106$ N/mm² $= 106$ MPa **Ans.**

Example 4.12. *A central steel rod 18 mm diameter passes through a copper tube 24 mm inside and 40 mm outside diameter, as shown in Fig. 4.15. It is provided with nuts and washers at each end. The nuts are tightened until a stress of 10 MPa is set up in the steel.*

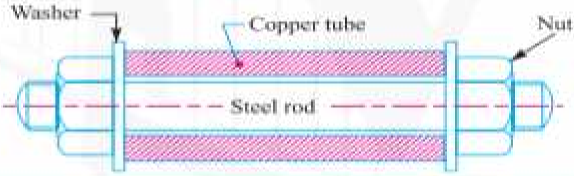


Fig. 4.15

The whole assembly is then placed in a lathe and turned along half the length of the tube removing the copper to a depth of 1.5 mm. Calculate the stress now existing in the steel. Take $E_s = 2E_c$.

Solution. Given : $d_s = 18$ mm ; $d_{c1} = 24$ mm ; $d_{c2} = 40$ mm ; $\sigma_s = 10$ MPa $= 10$ N/mm²

We know that cross-sectional area of steel rod,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(18)^2 = 254.5 \text{ mm}^2$$

and cross-sectional area of copper tube,

$$A_c = \frac{\pi}{4}\left[(d_{c2})^2 - (d_{c1})^2\right] = \frac{\pi}{4}\left[(40)^2 - (24)^2\right] = 804.4 \text{ mm}^2$$

We know that when the nuts are tightened on the tube, the steel rod will be under tension and the copper tube in compression.

Let σ_c = Stress in the copper tube.

Since the tensile load on the steel rod is equal to the compressive load on the copper tube, therefore

$$\sigma_s \times A_s = \sigma_c \times A_c$$

$$10 \times 254.5 = \sigma_c \times 804.4$$

$$\therefore \quad \sigma_c = \frac{10 \times 254.5}{804.4} = 3.16 \text{ N/mm}^2$$

When the copper tube is reduced in the area for half of its length, then outside diameter of copper tube,

$$= 40 - 2 \times 1.5 = 37 \text{ mm}$$

$\therefore$ Cross-sectional area of the half length of copper tube,

$$A_{c1} = \frac{\pi}{4}(37^2 - 24^2) = 623 \text{ mm}^2$$

The cross-sectional area of the other half remains same. If A_{c2} be the area of the remainder, then

$$A_{c2} = A_c = 804.4 \text{ mm}^2$$

Let σ_{c1} = Compressive stress in the reduced section,

σ_{c2} = Compressive stress in the remainder, and

σ_{s1} = Stress in the rod after turning.

Since the load on the copper tube is equal to the load on the steel rod, therefore

$$A_{c1} \times \sigma_{c1} = A_{c2} \times \sigma_{c2} = A_s \times \sigma_{s1}$$

$$\therefore \quad \sigma_{c1} = \frac{A_s}{A_{c1}} \times \sigma_{s1} = \frac{254.5}{623} \times \sigma_{s1} = 0.41\,\sigma_{s1} \quad \text{...(i)}$$

$$\text{and} \quad \sigma_{c2} = \frac{A_s}{A_{c2}} \times \sigma_{s1} = \frac{254.5}{804.4} \times \sigma_{s1} = 0.32\,\sigma_{s1} \quad \text{...(ii)}$$

Let δl = Change in length of the steel rod before and after turning,

l = Length of the steel rod and copper tube between nuts,

δl_1 = Change in length of the reduced section (*i.e.* $l/2$) before and after turning, and

δl_2 = Change in length of the remainder section (*i.e.* $l/2$) before and after turning.

Since $\delta l = \delta l_1 + \delta l_2$

$$\therefore \quad \frac{\sigma_s - \sigma_{s1}}{E_s} \times l = \frac{\sigma_{c1} - \sigma_c}{E_c} \times \frac{l}{2} + \frac{\sigma_{c2} - \sigma_c}{E_c} \times \frac{l}{2}$$

$$\text{or} \quad \frac{10 - \sigma_{s1}}{2E_c} = \frac{0.41\,\sigma_{s1} - 3.16}{2E_c} + \frac{0.32\,\sigma_{s1} - 3.16}{2E_c} \quad \text{...(Cancelling } l \text{ throughout)}$$

$$\therefore \quad \sigma_{s1} = 9.43 \text{ N/mm}^2 = 9.43 \text{ MPa} \quad \textbf{Ans.}$$

Example 4.13. *A thin steel tyre is shrunk on to a locomotive wheel of 1.2 m diameter. Find the internal diameter of the tyre if after shrinking on, the hoop stress in the tyre is 100 MPa. Assume $E = 200$ kN/mm². Find also the least temperature to which the tyre must be heated above that of the wheel before it could be slipped on. The coefficient of linear expansion for the tyre is 6.5×10^{-6} per °C.*

Solution. Given : $D = 1.2$ m = 1200 mm ; $\sigma = 100$ MPa = 100 N/mm² ; $E = 200$ kN/mm² $= 200 \times 10^3$ N/mm² ; $\alpha = 6.5 \times 10^{-6}$ per °C

Internal diameter of the tyre

Let d = Internal diameter of the tyre.

We know that hoop stress (σ),

$$100 = \frac{E\,(D-d)}{d} = \frac{200 \times 10^3\,(D-d)}{d}$$

$$\therefore \quad \frac{D-d}{d} = \frac{100}{200 \times 10^3} = \frac{1}{2 \times 10^3} \qquad \ldots(i)$$

$$\frac{D}{d} = 1 + \frac{1}{2 \times 10^3} = 1.0005$$

$$\therefore \quad d = \frac{D}{1.0005} = \frac{1200}{1.0005} = 1199.4 \text{ mm} = 1.1994 \text{ m} \quad \textbf{Ans.}$$

Least temperature to which the tyre must be heated

Let t = Least temperature to which the tyre must be heated.

We know that

$$\pi D = \pi d + \pi d\,.\,\alpha.t = \pi d\,(1 + \alpha.t)$$

$$\alpha.t = \frac{\pi D}{\pi d} - 1 = \frac{D-d}{d} = \frac{1}{2 \times 10^3} \qquad \ldots[\text{From equation } (i)]$$

$$\therefore \quad t = \frac{1}{\alpha \times 2 \times 10^3} = \frac{1}{6.5 \times 10^{-6} \times 2 \times 10^3} = 77\,°\text{C} \quad \textbf{Ans.}$$

Example 4.14. *A composite bar made of aluminium and steel is held between the supports as shown in Fig. 4.16. The bars are stress free at a temperature of 37°C. What will be the stress in the two bars when the temperature is 20°C, if (a) the supports are unyielding; and (b) the supports yield and come nearer to each other by 0.10 mm?*

It can be assumed that the change of temperature is uniform all along the length of the bar. Take $E_s = 210$ GPa ; $E_a = 74$ GPa ; $\alpha_s = 11.7 \times 10^{-6}$ / °C ; and $\alpha_a = 23.4 \times 10^{-6}$ / °C.

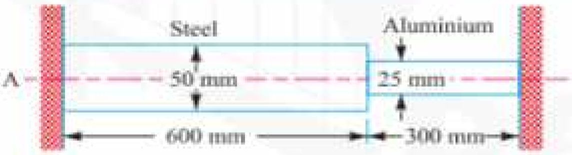


Fig. 4.16

Solution. Given : $t_1 = 37$°C ; $t_2 = 20$°C ; $E_s = 210$ GPa $= 210 \times 10^9$ N/m² ; $E_a = 74$ GPa $= 74 \times 10^9$ N/m² ; $\alpha_s = 11.7 \times 10^{-6}$ / °C ; $\alpha_a = 23.4 \times 10^{-6}$ / °C , $d_s = 50$ mm = 0.05 m ; $d_a = 25$ mm = 0.025 m ; $l_s = 600$ mm = 0.6 m ; $l_a = 300$ mm = 0.3 m

Let us assume that the right support at B is removed and the bar is allowed to contract freely due to the fall in temperature. We know that the fall in temperature,

$$t = t_1 - t_2 = 37 - 20 = 17\,°\text{C}$$

$\therefore$ Contraction in steel bar

$$= \alpha_s\,.\,l_s\,.\,t = 11.7 \times 10^{-6} \times 600 \times 17 = 0.12 \text{ mm}$$

and contraction in aluminium bar

$$= \alpha_a\,.\,l_a\,.\,t = 23.4 \times 10^{-6} \times 300 \times 17 = 0.12 \text{ mm}$$

Total contraction $= 0.12 + 0.12 = 0.24$ mm $= 0.24 \times 10^{-3}$ m

It may be noted that even after this contraction (*i.e.* 0.24 mm) in length, the bar is still stress free as the right hand end was assumed free.

Let an axial force P is applied to the right end till this end is brought in contact with the right hand support at B, as shown in Fig. 4.17.

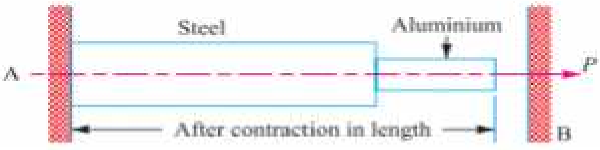


Fig. 4.17

We know that cross-sectional area of the steel bar,

$$A_s = \frac{\pi}{4}(d_s)^2 = \frac{\pi}{4}(0.05)^2 = 1.964 \times 10^{-3}\ \text{m}^2$$

and cross-sectional area of the aluminium bar,

$$A_a = \frac{\pi}{4}(d_a)^2 = \frac{\pi}{4}(0.025)^2 = 0.491 \times 10^{-3}\ \text{m}^2$$

We know that elongation of the steel bar,

$$\delta l_s = \frac{P \times l_s}{A_s \times E_s} = \frac{P \times 0.6}{1.964 \times 10^{-3} \times 210 \times 10^9} = \frac{0.6P}{412.44 \times 10^6}\ \text{m}$$

$$= 1.455 \times 10^{-9} P\ \text{m}$$

and elongation of the aluminium bar,

$$\delta l_a = \frac{P \times l_a}{A_a \times E_a} = \frac{P \times 0.3}{0.491 \times 10^{-3} \times 74 \times 10^9} = \frac{0.3P}{36.334 \times 10^6}\ \text{m}$$

$$= 8.257 \times 10^{-9} P\ \text{m}$$

∴ Total elongation, $\delta l = \delta l_s + \delta l_a$

$$= 1.455 \times 10^{-9} P + 8.257 \times 10^{-9} P = 9.712 \times 10^{-9} P\ \text{m}$$

Let σ_s = Stress in the steel bar, and

σ_a = Stress in the aluminium bar.

(a) When the supports are unyielding

When the supports are unyielding, the total contraction is equated to the total elongation *i.e.*

$$0.24 \times 10^{-3} = 9.712 \times 10^{-9} P \quad \text{or} \quad P = 24\,712\ \text{N}$$

∴ Stress in the steel bar,

$$\sigma_s = P/A_s = 24\,712 / (1.964 \times 10^{-3}) = 12\,582 \times 10^3\ \text{N/m}^2$$

$$= 12.582\ \text{MPa}$$ **Ans.**

and stress in the aluminium bar,

$$\sigma_a = P/A_a = 24\,712 / (0.491 \times 10^{-3}) = 50\,328 \times 10^3\ \text{N/m}^2$$

$$= 50.328\ \text{MPa}$$ **Ans.**

(b) When the supports yield by 0.1 mm

When the supports yield and come nearer to each other by 0.10 mm, the net contraction in length

$$= 0.24 - 0.1 = 0.14\ \text{mm} = 0.14 \times 10^{-3}\ \text{m}$$

Equating this net contraction to the total elongation, we have

$$0.14 \times 10^{-3} = 9.712 \times 10^{-9} P \quad \text{or} \quad P = 14\,415\ \text{N}$$

∴ Stress in the steel bar,

$$\sigma_s = P/A_s = 14\,415 / (1.964 \times 10^{-3}) = 7340 \times 10^3\ \text{N/m}^2$$

$$= 7.34\ \text{MPa}$$ **Ans.**

and stress in the aluminium bar,

$$\sigma_a = P/A_a = 14\,415 / (0.491 \times 10^{-3}) = 29\,360 \times 10^3\ \text{N/m}^2$$

$$= 29.36\ \text{MPa}$$ **Ans.**

Example 4.16. *A mild steel rod supports a tensile load of 50 kN. If the stress in the rod is limited to 100 MPa, find the size of the rod when the cross-section is 1. circular, 2. square, and 3. rectangular with width = 3 × thickness.*

Solution. Given : $P = 50\ \text{kN} = 50 \times 10^3\ \text{N}$; $\sigma_t = 100\ \text{MPa} = 100\ \text{N/mm}^2$

1. Size of the rod when it is circular

Let d = Diameter of the rod in mm.

∴ Area, $A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 0.7854\, d^2 = 78.54\, d^2$$

∴ $d^2 = 50 \times 10^3 / 78.54 = 636.6$ or $d = 25.23$ mm **Ans.**

2. Size of the rod when it is square

Let x = Each side of the square rod in mm.

∴ Area, $A = x \times x = x^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times x^2$$

∴ $x^2 = 50 \times 10^3/100 = 500$ or $x = 22.4$ mm **Ans.**

3. *Size of the rod when it is rectangular*

Let t = Thickness of the rod in mm, and

b = Width of the rod in mm = $3t$ ...(Given)

$\therefore$ Area, $A = b \times t = 3t \times t = 3t^2$

We know that tensile load (P),

$$50 \times 10^3 = \sigma_t \times A = 100 \times 3t^2 = 300t^2$$

$$\therefore \quad t^2 = 50 \times 10^3 / 300 = 166.7 \quad \text{or} \quad t = 12.9 \text{ mm} \quad \textbf{Ans.}$$

and $b = 3t = 3 \times 12.9 = 38.7$ mm **Ans.**

Example 4.17. *A steel bar 2.4 m long and 30 mm square is elongated by a load of 500 kN. If poisson's ratio is 0.25, find the increase in volume. Take $E = 0.2 \times 10^6$ N/mm^2.*

Solution. Given : $l = 2.4$ m = 2400 mm ; $A = 30 \times 30 = 900$ mm^2 ; $P = 500$ kN = 500×10^3 N ; $1/m = 0.25$; $E = 0.2 \times 10^6$ N/mm^2

Let δV = Increase in volume.

We know that volume of the rod,

$$V = \text{Area} \times \text{length} = 900 \times 2400 = 2160 \times 10^3 \text{ mm}^3$$

and Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\varepsilon}$$

$$\therefore \quad \varepsilon = \frac{P}{A.E} = \frac{500 \times 10^3}{900 \times 0.2 \times 10^6} = 2.8 \times 10^{-3}$$

We know that volumetric strain,

$$\frac{\delta V}{V} = \varepsilon\left(1 - \frac{2}{m}\right) = 2.8 \times 10^{-3} \ (1 - 2 \times 0.25) = 1.4 \times 10^{-3}$$

$$\therefore \quad \delta V = V \times 1.4 \times 10^{-3} = 2160 \times 10^3 \times 1.4 \times 10^{-3} = 3024 \text{ mm}^3 \ \textbf{Ans.}$$

Example 4.19. *A wrought iron bar 50 mm in diameter and 2.5 m long transmits a shock energy of 100 N-m. Find the maximum instantaneous stress and the elongation. Take E = 200 GN/m^2.*

Solution. Given : $d = 50$ mm ; $l = 2.5$ m = 2500 mm ; $U = 100$ N m = 100×10^3 N mm ; $E = 200$ GN/m^2 = 200×10^3 N/mm^2

Maximum instantaneous stress

Let σ = Maximum instantaneous stress.

We know that volume of the bar,

$$V = \frac{\pi}{4} \times d^2 \times l = \frac{\pi}{4}(50)^2 \times 2500 = 4.9 \times 10^6 \text{ mm}^3$$

We also know that shock or strain energy stored in the body (U),

$$100 \times 10^3 = \frac{\sigma^2 \times V}{2E} = \frac{\sigma^2 \times 4.9 \times 10^6}{2 \times 200 \times 10^3} = 12.25\,\sigma^2$$

$$\therefore \quad \sigma^2 = 100 \times 10^3 / 12.25 = 8163 \quad \text{or} \quad \sigma = 90.3 \text{ N/mm}^2 \ \textbf{Ans.}$$

Elongation produced

Let δl = Elongation produced.

We know that Young's modulus,

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\delta l / l}$$

$$\therefore \quad \delta l = \frac{\sigma \times l}{E} = \frac{90.3 \times 2500}{200 \times 10^3} = 1.13 \text{ mm} \ \textbf{Ans.}$$

Example 5.5. *A steel shaft ABCD having a total length of 3.5 m consists of three lengths having different sections as follows:*

AB is hollow having outside and inside diameters of 100 mm and 62.5 mm respectively, and BC and CD are solid. BC has a diameter of 100 mm and CD has a diameter of 87.5 mm. If the angle of twist is the same for each section, determine the length of each section. Find the value of the applied torque and the total angle of twist, if the maximum shear stress in the hollow portion is 47.5 MPa and shear modulus, C = 82.5 GPa.

Solution. Given: $L = 3.5$ m ; $d_o = 100$ mm ; $d_i = 62.5$ mm ; $d_2 = 100$ mm ; $d_3 = 87.5$ mm ; $\tau = 47.5$ MPa = 47.5 N/mm^2 ; $C = 82.5$ GPa = 82.5×10^3 N/mm^2

The shaft $ABCD$ is shown in Fig. 5.3.

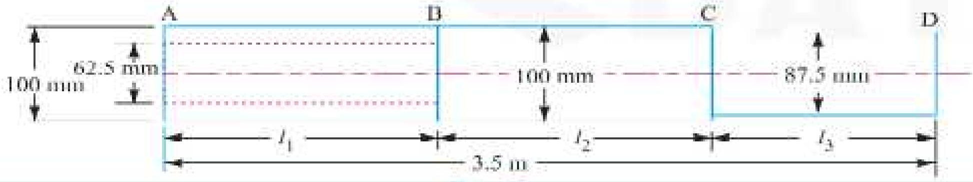


Fig. 5.3

and polar moment of inertia of the solid shaft CD,

$$J_3 = \frac{\pi}{32}(d_3)^4 = \frac{\pi}{32}(87.5)^4 = 5.75 \times 10^6 \text{ mm}^4$$

We also know that angle of twist,

$$\theta = T \cdot l / C \cdot J$$

Assuming the torque T and shear modulus C to be same for all the sections, we have

Angle of twist for hollow shaft AB,

$$\theta_1 = T \cdot l_1 / C \cdot J_1$$

Similarly, angle of twist for solid shaft BC,

$$\theta_2 = T \cdot l_2 / C \cdot J_2$$

and angle of twist for solid shaft CD,

$$\theta_3 = T \cdot l_3 / C \cdot J_3$$

Since the angle of twist is same for each section, therefore $\theta_1 = \theta_2$

$$\frac{T \cdot l_1}{C \cdot J_1} = \frac{T \cdot l_2}{C \cdot J_2} \text{ or } \frac{l_1}{l_2} = \frac{J_1}{J_2} = \frac{8.32 \times 10^6}{9.82 \times 10^6} = 0.847 \quad ...(i)$$

Also $\theta_1 = \theta_3$

$$\frac{T \cdot l_1}{C \cdot J_1} = \frac{T \cdot l_3}{C \cdot J_3} \text{ or } \frac{l_1}{l_3} = \frac{J_1}{J_3} = \frac{8.32 \times 10^6}{5.75 \times 10^6} = 1.447 \quad ...(ii)$$

We know that $l_1 + l_2 + l_3 = L = 3.5 \text{ m} = 3500 \text{ mm}$

$$l_1\left(1 + \frac{l_2}{l_1} + \frac{l_3}{l_1}\right) = 3500$$

$$l_1\left(1 + \frac{1}{0.847} + \frac{1}{1.447}\right) = 3500$$

$$l_1 \times 2.8717 = 3500 \text{ or } l_1 = 3500 / 2.8717 = 1218.8 \text{ mm } \textbf{Ans.}$$

From equation *(i)*,

$$l_2 = l_1 / 0.847 = 1218.8 / 0.847 = 1439 \text{ mm } \textbf{Ans.}$$

and from equation *(ii)*, $l_3 = l_1 / 1.447 = 1218.8 / 1.447 = 842.2$ mm **Ans.**

Value of the applied torque

We know that the maximum shear stress in the hollow portion,

$$\tau = 47.5 \text{ MPa} = 47.5 \text{ N/mm}^2$$

For a hollow shaft, the applied torque,

$$T = \frac{\pi}{16} \times \tau\left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times 47.5\left[\frac{(100)^4 - (62.5)^4}{100}\right]$$

$$= 7.9 \times 10^6 \text{ N-mm} = 7900 \text{ N-m } \textbf{Ans.}$$

Total angle of twist

When the shafts are connected in series, the total angle of twist is equal to the sum of angle of twists of the individual shafts. Mathematically, the total angle of twist,

$$\theta = \theta_1 + \theta_2 + \theta_3$$

$$= \frac{T \cdot l_1}{C \cdot J_1} + \frac{T \cdot l_2}{C \cdot J_2} + \frac{T \cdot l_3}{C \cdot J_3} = \frac{T}{C}\left[\frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3}\left[\frac{1218.8}{8.32 \times 10^6} + \frac{1439}{9.82 \times 10^6} + \frac{842.2}{5.75 \times 10^6}\right]$$

$$= \frac{7.9 \times 10^6}{82.5 \times 10^3 \times 10^6}[146.5 + 146.5 + 146.5] = 0.042 \text{ rad}$$

$$= 0.042 \times 180 / \pi = 2.406° \textbf{ Ans.}$$

Example 5.6. *A pump lever rocking shaft is shown in Fig. 5.5. The pump lever exerts forces of 25 kN and 35 kN concentrated at 150 mm and 200 mm from the left and right hand bearing respectively. Find the diameter of the central portion of the shaft, if the stress is not to exceed 100 MPa.*

Fig. 5.5

Solution. Given : $\sigma_b = 100 \text{ MPa} = 100 \text{ N/mm}^2$

Let R_A and R_B = Reactions at A and B respectively.

Taking moments about A, we have

$$R_B \times 950 = 35 \times 750 + 25 \times 150 = 30\,000$$

$$\therefore \quad R_B = 30\,000 / 950 = 31.58 \text{ kN} = 31.58 \times 10^3 \text{ N}$$

and $R_A = (25 + 35) - 31.58 = 28.42 \text{ kN} = 28.42 \times 10^3$ N

$\therefore$ Bending moment at C

$$= R_A \times 150 = 28.42 \times 10^3 \times 150 = 4.263 \times 10^6 \text{ N-mm}$$

and bending moment at $D = R_B \times 200 = 31.58 \times 10^3 \times 200 = 6.316 \times 10^6$ N-mm

We see that the maximum bending moment is at D, therefore maximum bending moment, $M = 6.316 \times 10^6$ N-mm.

Let d = Diameter of the shaft.

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

We know that bending stress (σ_b),

$$100 = \frac{M}{Z} = \frac{6.316 \times 10^6}{0.0982\, d^3} = \frac{64.32 \times 10^6}{d^3}$$

$\therefore$ $d^3 = 64.32 \times 10^6/100 = 643.2 \times 10^3$ or $d = 86.3$ say 90 mm **Ans.**

Example 5.7. *An axle 1 metre long supported in bearings at its ends carries a fly wheel weighing 30 kN at the centre. If the stress (bending) is not to exceed 60 MPa, find the diameter of the axle.*

Solution. Given : $L = 1$ m = 1000 mm ; $W = 30$ kN = 30×10^3 N ; $\sigma_b = 60$ MPa = 60 N/mm²

The axle with a flywheel is shown in Fig. 5.6.

Let d = Diameter of the axle in mm.

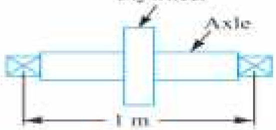


Fig. 5.6

$\therefore$ Section modulus,

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$$

Maximum bending moment at the centre of the axle,

$$M = \frac{W.L}{4} = \frac{30 \times 10^3 \times 1000}{4} = 7.5 \times 10^6 \text{ N-mm}$$

We know that bending stress (σ_b),

$$60 = \frac{M}{Z} = \frac{7.5 \times 10^6}{0.0982\, d^3} = \frac{76.4 \times 10^6}{d^3}$$

$\therefore$ $d^3 = 76.4 \times 10^6/60 = 1.27 \times 10^6$ or $d = 108.3$ say 110 mm **Ans.**

Example 5.8. *A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.*

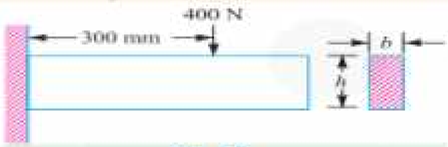


Fig. 5.7

Solution. Given: $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa = 40 N/mm² ; $h = 2b$

The beam is shown in Fig. 5.7.

Let b = Width of the beam in mm, and

h = Depth of the beam in mm.

$\therefore$ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress (σ_b),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

$\therefore$ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

Example 5.9. *A cast iron pulley transmits 10 kW at 400 r.p.m. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.*

Solution. Given : $P = 10$ kW = 10×10^3 W ; $N = 400$ r.p.m ; $D = 1.2$ m = 1200 mm or $R = 600$ mm ; $\sigma_b = 15$ MPa = 15 N/mm²

Let T = Torque transmitted by the pulley.

We know that the power transmitted by the pulley (P),

$$10 \times 10^3 = \frac{2\,\pi\, N \cdot T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\, T$$

$\therefore$ $T = 10 \times 10^3/42 = 238$ N-m = 238×10^3 N-mm

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm = $2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}\,(2b)^2 \times b = \pi\, b^3 \text{ mm}^3$$

We know that bending stress (σ_b),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\, b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**